Basics of

Quantitative Ability

Contents:

Number Systems

1 is the smallest Natural number, 0 is the smallest Whole number, and there is no largest or smallest Integer.

Even numbers are multiples of 2. Any even number can be written as 2n, where n is an integer. ***0 is an even number***.

Odd numbers are numbers which when divided by 2 leave a remainder of 1. Any odd number can be written as 2n +1, where n is an integer.

Factors of a given natural number say n, is another natural number say ,f if n is completely divisible by f. Ex factors of 18 are 1, 2,3,6,9,18 and number of factors is 6.

Highest factor of any natural number is number itself and lowest positive factor is 1. 1 is the factor of every natural number.

The number of factors of any natural number is finite.

A natural number which has exactly 2 factors is a prime number. Ex number 2 has factors 1 and 2 only. Similarly 3,5,7,11,13,17,19,23,29,31,37,41 etc ***. 1 is not a prime number.***

A number n is a prime number if it is not divisible by any prime less than [√n] where [√n] is the largest natural number less than or equal to √n.

Any natural number n can be written in the form and in one unique way only $n = p^{\alpha} \times q^{\beta} \times r^{\gamma}$ ….., where p,q,r… are different primes and α,β,γ…. Are the powers of the prime number respectively.

Example

$18 = 2^1 \times 3^2$. Here 2 and 3 are the prime numbers and 1 and 2 are the respective powers of the primes.

The number of factors of any natural number n, which can be factored as above is = (1+α)x(1+β)x(1+γ)…. etc. Thus number of factors of 18 is (1+1) x (1+2) = 6.

The number of odd factors will be given as follows. If the given number does not have any power of 2 then number of factors is same as number of odd factors. But if the number has any term like 2^{α} in its factorization, as product of prime number powers , where α ≥ 1, then exclude the (1+α) term and calculate the number of odd factors as (1+β)x(1+γ)…. Etc. Thus number of odd factors of 18 is (1+2) = 3, viz... 1, 3 and 9.

The sum of all the factors is given by the expression $(p^{\alpha-1}-1)\times(q^{\beta-1}-1)\times(r^{\gamma-1}-1)/((p-1)\times(q-1)\times(r-1)$…..

Composite number is a number which has more than 2 factors. For example number 18 has 6 factors viz. 1,2,3,6,9,18.

Remainder

Any whole number say m is divided by another natural number say n then there exists numbers q and r such that m = nxq + r. Where **q** is known as the **quotient** and **r** is known as the **remainder**. For any m ,n ? N(the set of natural numbers) q and r ? W(the set of whole numbers). Thus 0 ≤r

Class of integers

As discussed above the remainder obtained when any number is divided by say 5 then remainder is either 0 ,1,2,3,4 only. Therefore any number can be written as either as 5k , 5k+1 , 5k+2 , 5k+3 , 5k+4. Thus entire set of numbers has been split into 5 non overlapping sets.

HCF

HCF of any given set of numbers is a number which completely divides each number in the given set and the number is highest such number possible.

LCM

LCM of any given set of numbers is the smallest such number which is divisible by each number of the given set.

For any 2 given numbers HCFxLCM = Product of the 2 numbers.

HCF of fractions = HCF of numerators of all the given fractions/LCM of the denominators of all the fractions.

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Divisibility Rules

If the last digit of a number is even then number is divisible by 2

If the sum of the digits of a number is divisible by 3 then 9 then number is divisible by 3 and 9 respectively.

If the last 2 digits of the number are divisible by 4 then number is divisible by 4 and if last 3 digits is divisible by 8 then number is divisible by 8.

If the last digit of the number is 0 or 5 then number is divisible by 5

If the sum of digits of the number is divisible by 3 and the last digit is even then number is divisible by 6.

A given number is divisible by 7 if the number of tens in the original number - twice the units digits is divisible by 7 ex. to check whether 343 is divisible by 7 or not , Thus twice the units digit is 2x3=6 and number of tens in the number is 34. Therefore 343 is divisible by 7 if 34- 6 is divisible by 7 i.e. 28 is divisible by 7.

A number is divisible by 11 if the difference of the sum of digits occurring the even numbered places and the sum of digits occurring in the odd number of places is divisible by 11.

Surds and Indices

Rules of Indices

$b^{l} x b^{m} x b^{n} .....= b^{l+m+p.....}$

$(p^{m})^{n} = p^{mn}$

If bases are same then powers are also same. i.e. if $a^{m} = a^{p}$ and if a ≠1 or 0 then it implies m = p.

The last digit of the square of any number cannot be 2,3,7 or 8.

Any perfect square is of the form either 4k or 4k-1 or 4k+1.

Product of any number of even numbers is even and any number of odd numbers is odd.

The product of any n consecutive natural numbers is divisible by n!.

Useful Algebraic identities

$(a^{n} + b^{n})$ is divisible by (a+b) for all odd values of n.

$(a^{n} - b^{n})$ is divisible by both (a+b) and (a-b) for even values of n.

$(a^{n} - b^{n})$ is divisible by (a-b) for all values of n (both odd and even).

Sum of first n natural numbers = n(n+1)/2.

Sum of squares of first n natural numbers = n(n+1)(2n+1)/6

Sum of the cubes of first n natural numbers is given by $\{nx(n+1)/2\}^{2}$.

Units digit of any power of a number (Cyclicity)

If we consider the units digit of the powers of 2 i.e. 2^{x} for different values of x then we find the unit's digit is 2,4,6,8,2,4,6,8,2,4,6,8.... for x= 1,2,3,4,5,6,7,8,9,10,11,12.... Similar patterns exist for the unit's digit of other numbers. The results are summarized in the table below.

	Unit's digit of a^x, k is an natural number			
Number "a"	x=4k+1	x=4k+2	x=4k+3	x=4k
1	1	1	1	1
2	2	4	6	8
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

Solved Examples

QUESTION

The Rightmost non zero digit of 5670^{5670} is

(a) 7 (b) 1 (c) 3 (d) 9

SOLUTION

Here we use rule of cyclicity to solve this problem. The first non zero digit to the left of 0 is 7 hence the rightmost non zero digit will be same as the units digit of 7^{5670}. Now next we need to find out the form of 5670. Dividing 5670 by 4 we get 2 as remainder. Hence 5670 is of the form 4k +2. Therefore the unit's digit is 9. Therefore answer is d

QUESTION

The numbers 13409 and 16760 when divided by a 4 digit integer n leave the same remainder then the value of n is

(a) 1127 (b)1117 (c) 1357 (d)1547

SOLUTION

Here 13409 and 16760 on division by n leave the same remainder hence can be written as nxm + r and can be written nxp + r. Therefore subtracting the 2 equations we get nx(m-n) =3351 = 1117x3 But n is 4 digit number hence is n is 1117. Hence answer is b.

QUESTION

If N = $82^3 – 62^3 - 20^3$ then N is divisible by

(a) 41 and 31 (b) 13 and 67 (c) 17 and 7 (d) None of these

SOLUTION

Using the identity $a^3 + b^3 + c^3 - 3abc = (a+ b+ c)x(a^2 + b^2 + c^2 – ab - ac – ac)$. We find that if a + b + c = 0. Then $a^3 + b^3 + c^3 = 3abc$. Therefore $a^3 + b^3 + c^3$ is divisible by a , b , c and 3. Now a = 82 , b = -62 and c = -20. Therefore N is divisible by 3 , 82 , 62 and 20. But 82 is divisible by 41 and 31. Hence answer is a.

QUESTION

Let X be a set of positive integers such that every element a of S satisfies the condition (i) $1100 \le n \le 1300$ (ii) every digit in n is odd

Then how many elements of X are divisible by 3?

(a) 10 (b) 9 (c) 16 (d) 13

SOLUTION

We know that any number is divisible by 3 if the sum of the digits of the number is divisible by 3. Now between 1200 and 1299(both inclusive) we find that all the numbers have one digit viz. 2(the hundreds digit) which is even, hence there do not exists any number between 1200 and 1299 with the given conditions. Also 1300 is not divisible by 3. Therefore the required number will lie between 1100 and 1199. Between 1100 and 1199 all the numbers will have first 2 digit as 1 Therefore if we consider any number between 1100 and 1199 it will be of the form 11ab , where a, b are digits. Sum of digits will be given by 1 +1 +a +b i.e. 2+a+b. Now a, b are digits therefore maximum value of a +b can be 18 hence max value of 2+a+b can be 20. Therefore for 11ab to be divisible by 3 value of 2+a + b can be either 18 , 15, 12, 9 , 6 or 3. Also a,b has to be odd ,therefore a +b will be even. Hence 2+ a +b will be even. Hence 2 + a +b can assume values which are even. Therefore 2 + a+ b can be either 18 or 12 or 6 only. Which => a+b can be 16, 10, and 4 only. Solving a +b =16 for odd a, b we get a=9, 7 and b = 7, 9 respectively. Similarly solving a +b = 12 we get a = 9, 7, 5,3 and b = 3,5,7,9 respectively and solving a + b =6 we get a = 5 , 3 ,1 and b = 1 ,3,5 respectively so total number of solution is 9.

QUESTION

Four bells toll at intervals of 15, 18, 24 and 32 minutes respectively. At a certain time , they begin to toll together after what , least time interval of time(in mins) will they toll together again?

(a) 1440 (b) 900 (c) 1660 (d) 1335

SOLUTION

The first bell tolls at after every 15 minutes i.e. at 15 minutes , 30 minutes , 45 minutes etc. Similarly the second bell will toll at 18 minutes , 36 minutes etc, Thus we observe that all the 4 bells will toll together at a time which is common multiple of 15 , 18 , 24 and 32. Also as we are looking for least such interval hence the required time is the LCM of 15 , 18 , 24 and 32 which is 1440 mins

Percentages, Profit & Loss

A student get 20 marks out of 30 in an examination , then we express this information as follows . If the student got 20 marks when the total marks were 30 what would he have got if the total marks was 100?

The amount to this question is $\frac{20}{30}\times 100$ = $\frac{200}{3}$ = 66.66% .
The person of finding the answer to the question what is the value if total in 100 is what leads is to percentage . This if we say student scored 66.66 percent it mean if total is 100 then student got 66.66 marks.

Equivalence between fraction and percentage

Fraction	Equivalence Percentage
1	100%
1/2	50%
1/3	33.33%
1/4	25%
1/5	20%
1/6	16.67%
1/7	14.28%
1/8	12.5%
1/9	11.11%
1/11	9.09%

Percentage Change

If a quantity A is increased by 'x' then percentage increase = $\frac{change}{original}\times 100 = \frac{(a-x)-a}{a}\times 100$
= $\frac{100x}{A}$ %

If a quantity is increased by a% then new value is given by $\left(1+\frac{a}{100}\right)\times original quantity$ similar if a quantity is decreased by a% then new value is given by $\left(1-\frac{a}{100}\right)\times original quantity$.

If a quantity is increased by a% and then further by b% these percent change given are equivalent to a single percent change given by

$\left(a+b+\frac{ab}{100}\right)$ %

Similarly if there is a successive decrease of a% followed by b% then effective percentage decrease is

$\left(a+b-\frac{ab}{100}\right)$

Selling Price, Marked Price and Cost Price

Selling price

(SP) is the price at which an article is sold cost price is the price at which an article is bought. If SP > CP then the difference SP – CP is known as the profit , P P = SP – CP

and if SP < CP then the difference CP – SP is known as loss, L L = CP – SP

Profit Percentage P% = $\frac{profit}{CP}\times 100$

Loss Percentage L% = $\frac{oss}{CP}\times 100$

Marked price.

The price which is displayed on the tag of the article is known as marked price.

Generally the SP is less then the marked price (MP) the difference MP – SP is known as discount D

Discount % , D% = $\frac{Discount}{MP}\times 100$

Solved Examples

Question

A shopkeeper sells sugar for Rs 25 per kg and makes a profit of 20%. If he sells the sugar for Rs 22.50 per Kg find his profit ?

Solution

Let the cost price CP of sugar be rs x per Kg then SP = CP + Profit

But Profit = $\frac{Profit\times CP}{100}=\frac{20}{100}\times CP$

SP = $CP\left(1+\frac{20}{100}\right)$

25 = CP (1.2)

CP = $\frac{25}{1.2}=20.83\,Rs/kg$

Hence profit when SP is 22.50 Rs/ Kg

= 1.67 Rs/Kg

Profit % = $\frac{1.67}{20.83}\times 100=8$ %

Question

Mango bite is available for Rs 8 a dozen . If the shopkeeper offer a discount of 10% how many mango bites can be purchased for Rs 2.4

Solution

Discount Percentage = 10%

Marked price for 12 mango bites = Rs 8

8P of 12 mango bites = MP – Discount

Discount = $\frac{D\times MP}{100}=\frac{10\times 8}{100}=.8$

Hence SP = $8\times .3=7.2$

12 toffees are available for Rs 7.2 hence for 2.4 we get 4 toffees.

Question

A men sells 2 pens Rs 100 each at a profit of 10% and another at a loss of 10% find his overall project or loss percentage.

Solution

If the cost price of the 2 pens be x an y respectively then using profit % $\frac{SP-CP}{CP}\times 100$ we get

CP = $\frac{100\times P}{100+P}$ in case of project

and CP = $\frac{100\times L}{100-L}$ in case of loss

Hence x = $\frac{100\times 10}{100+10}$ and y = $\frac{100\times 10}{100-10}$

x = $\frac{100}{11}$ and y $\frac{100}{9}$

Total CP of both pens = $\frac{100}{11}+\frac{100}{9}=\frac{2000}{99}$

Total loss = $\frac{2000}{99}-200$

= $\frac{2000-1980}{99}$

= $\frac{20}{99}$

Loss % = $\frac{\frac{20}{99}}{\frac{2000}{99}}\times 100$

= 1%

Note

In case 2 article are sold at same selling price one at a profit of a% another are at a loss of a% then there is overall loss on the whole out lay and loss percentage is given by

$\frac{a^2}{100}$ %

Question

20 is 16% of what number?

Solution

If number is 100 then 16% of 100 is 16 so if 20 is 16% of some number then

number = $\frac{20\times 100}{16}=125$

Question

A company decided to reduce the price of its product by 20% . By what percentage the sales volume should increase , so that the total revenues of the company remains unchanged ?

Solution

Revenues(R) = $Price(P) \times volume(V)$

R = $P \times v$ .... (1)

New price = $.8 \times P$

Hence if ' V^1 ' is the new volume then

$V^1 \times .8P = R$ .... (2) equating (1) and (2)

We get $PV = .8P \times V^1$

= $V^1 = \frac{V}{.8}$

= $\frac{5}{4}V$

= $\frac{5V}{4} - V = \frac{V}{4}$

Therefore percentage increased is volume

= $\frac{\frac{V}{4}}{V} \times 100 = 25$ %

Averages, Mixtures and Alligations

In arithmetic mean or the average of n quantities $x_1 + x_2 + x_3 \ldots\ldots\ldots x_n$ is given by

$$x = \frac{x_1 + x_2 - \ldots\ldots + x_n}{n}$$

The average is ≥ the smallest of $x_1 + x_2 + \ldots\ldots\ldots x_n$

The average is ≤ the greater of $x_1 + x_2 + \ldots\ldots\ldots + x_n$

The Sum of deviation of each element β with respect to AM is equal to zero i.e

$(x1 \; x) + (x2 - x) + \ldots\ldots\ldots\ldots (xn - x) = 0$

Rules of allegation

A shopkeeper mixes two verities of sugar costing Rs 25/Kg and Rs 20/Kg in a certain ratio such that the cost of the mixture is Rs 23/Kg then find the ratio in which the 2 types of sugar were mixed ?

Let the ratio in which sugar of rs 25/Kg and Rs 20/Kg be p:q then total cost of mixture = $\frac{25p + 20q}{p + q} = 23$

Solving for p and q we get $\frac{p}{q} = \frac{23 - 20}{25 - 23}$

This is the allegation rule.

Formally stated it states the ratio of dean quantity and the cheap quantity is equal to the ratio of the difference of the mean price and the cheaper price and the difference of the dean price and the mean

price symbolically $\frac{p}{q} = \frac{Mp - Cp}{Dp - Mp}$ where cp- cheaper price

Mp – mean price, Dp – Dean price

Mixtures

When we mix two or more then 2pure substance , we get what is known as a mixture.

Mixture of Mixtures :- If we mix two mixtures which have components say A and B in the final mixture is given by

$\frac{a}{b}$ = A

The weight of first and second mixture respectively.

A special problem on mixtures

If a contains 'x' liter of liquid 'x' and if 'y' liters is taken out and replaced by 'y' liters of liquid 'z' and if the above step is repeated with the mixture so obtained that is x liter of mixture replaced with y liter of liquid 'z' then if the above operation is repeated 'n' times then

$$\frac{\text{liquid X left the container after } n^{n} \text{ operation}}{\text{Initial quantity of liquid X the contais}} = \left(\frac{x - y}{x}\right)^{n}$$

$$\frac{\text{Amount Liquid Z the solution after } n^{n} \text{ operation}}{\text{initial quantity of liquid X the contains}} = 1 - \left(\frac{x - y}{x}\right)^{n}$$

Note: The final volume of mixture remains contents and is same as initial volume same as initial of liquid X

Solved Examples

Question

The average age of 10students in a class is increased by 2 year when two students aged 12 year and 14 year are replaced by 2girls . Find the average age of the two girls.

Solution

Let the sum of the ages of the 2girls be y years, also sum of the ages of the 2 students who have replaced = 26 years average age of group has gone up by 2 years that means the increase in total age of the 10 students is 20 years this increase is due to the age of the 2 girls hence y – 26 = 20 , y= 4b therefore average age of the 2 girls

Question

There are 60 students in a class . There students are divided into there groups A,B and c of 15, 20 an 25 students each . These groups A and C are combined to form group D.What is the average weight of the students in group D?

(1). More then the average weight of A

(2). More then the average weight of C

(3)Less then the average weight of C

(4) Cannot be determined.

Solution

Number of students in group D is more then number of students in group A or group C. But there is no information about the weight of students in group A or group C. Hence answer is (4)

Note : As a group D has students form group C whichever group has higher average weight , the average weight of group D will be that group s average weight

Question

If one student from group A is shifted group B, which of the following will be true?

(1) The average weight of all the four groups is same .

(2) The average weight of both the groups decrease.

(3) The average weight of the clear remains same

(4) Cannot to be determined.

Solution

Shifting students form groups A to group b , the total number of students in the clean remains same , hence

(3)is the correct answer.

Question

If all the students of the class have the same weight , then which of the following is not true?

(1) The average weight of all the four groups is same.

(2) The total weight of A and C is twice the total weight of B.

(3) The average weight of D is greater then the average weight of A.

(4)The average weight of all the groups remains the same even if a number of students are shifted from one group to another

Solution

Weight of each students is same is same therefore average weight of clean is same as average weight of group A,B,C and D.

Hence (3) cannot be true.

Hence option (3) is correct answer.

Time and Work

If a person A can do a piece of work in 'a' days, then working at the same uniform speed A will do $\frac{1}{a}$ fraction of the work in one day.

For example,

Days taken to complete 'a' work	25	12.5	1/2
Fraction of work done in a day	1/25 th	1/12.5 th	2

Working together – If working alone can do some work in 'a' days and if another person B working alone can do it in 'b' days , then if A and B

start working together the amount of work done by A & B in one day = $\frac{1}{a}+\frac{1}{b}$

Therefore total work is done by A and B working together in $\frac{ab}{a+b}$ days

Note: Remember it is work done that is additive and not number of days.

The LCM technique

If A completes a work in 'a' days working alone and B completes it in 'b' days working alone, then intended of assuming the work as 1 unit we can assume total work to be LCM of (a,b) this will avoid the fraction in the calculation.

Proportionality (concept of efficiency)

If A is thrice as efficient as B then if A takes 12 days to complete a work then B will take 12×3 = 36 days to complete the work . Whenever

relative efficiency of team member is given the most suitable approach is to consider the capacities of each team member in terms of any given

team member.

Amount of Work done is directly proportional number of person employed to do the work and days worked.

If amount of work is constant then number of days taken to complete the work is inversely proportional to days taken.

Men , Women and children working in a given project.

Solved Examples

Question

If work can be done in 10 days by 4 men or 6 women or 10 children. How many days are required for 3 men , 8 women and 6 days to complete it?

Solution

1 Man will take 40 days , hence 3 men will take 40/3 days to complete it

1 women will take 60 days, hence 8 women will take 60/8 days

1 child will take 100 days , hence 6 days will take 100/6 days

then total time taken if they all worked together is = $\frac{1}{\frac{3}{40}+\frac{8}{60}+\frac{6}{100}}=\frac{600}{45+80+36}=\frac{600}{161}$ days.

Question

A house can be built by A in 50 days and B can demolish it in 60 days . If A and B work on alternative days , in how many days will the house be built , assuming A stars first and once house is built completely B days not demolish it?

Solution

In 2 days cycle amount of work done = $\frac{1}{50}-\frac{1}{60}=\frac{1}{300}$ but does that mean work gets done in 600 days . Think again as a finisher $\frac{1}{50}$ th of

the work in one days , hence if work done is less then $\frac{1}{50}$ then A finisher the work and B does not demolish it .

In two days amount of work done is $\frac{1}{300}$ th therefore $\frac{49}{50}$ th fraction of the work is finished in

588 days work left to be done on 589^{th} days is $\frac{1}{50}$ which A will finish. Hence work gets finished in 589 days.

Time and Distance

Speed

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Speed is the rate at which distance is covered by a moving body these speed

$$'s' = \frac{\text{distance}(d)}{\text{time taken}(t)} \text{ or}$$

$$s = \frac{d}{t}$$

if speed is constant then $d \propto t$

if distance is constant then $s \propto \frac{1}{t}$

if time is constant then $d \propto s$

Average speed

Average speed. If a body cover half of the distance 'd' it a speed of s_1 and the half at speed s_2 then average speed is given by

$$\frac{\text{total distance covered}}{\text{total time taken}} = \frac{2d}{\frac{s_1}{d} + \frac{s_2}{d}} = \frac{2s_1 s_2}{s_1 + s_2}$$

Relative speed

The time taken by a train of length 'l' moving with speed 's' to pare a pole is

$$= \frac{l}{s}$$

Time taken by train of length 'l' moving with speed 's' to completely pare a platform of length 'd' = $\frac{l+d}{s}$

Time taken by strain of length 'l' moving with speed s_1 to par another train of length 'd' moving with speed s_2 in the direction (provided $s_1 > s_2$) = $\frac{l+d}{s_1 - s_2}$

Time taken by a train, of length l, and moving with speed s_1 to pan another train of length 'd' moving with s_2 in the opposite direction

$$\frac{l+d}{s_1 + s_2}$$

If person covers a certain distance 'd' moving speed S_1 and taking time t_1, then the person covers the same distance moving at speed S_2

and taking time t_2 then $\frac{S_1}{S_2} = \frac{t_1}{t_2}$

If two persons X and Y start at the same time in opposite direction from two places and arrive at their respective destination in x an y hours later after having met on their on their way then

$$\frac{\text{speed of x}}{\text{speed of y}} = \frac{\sqrt{y}}{\sqrt{x}}$$

If speed of boat, in still water is 'a' (m/s) then speed of boat in going down stream in a river which is following at 'b' (m/s) is (a+b) m/s. Hence if a boat cover a certain distance 'd' downstream and then reruns it to its staring point covering the same distance 'd' then

$$\frac{\text{upstream journey time}}{\text{downstream journey time}} = \frac{a+b}{a-b}$$

Circular races

If 2 persons say a,b start running a race on a circular track, then the faster runner (say A) will be together with the slower runner for the
first time when faster runner has gained one complete length of the track or round over the slower runner.

Solved Examples

Solved Examples

Question

2boats A and B start from point P at the same constant speed of 15 km/hr in still water . Boat A goes to Q, where boat B goes till R, which is equidistant from P and Q and returns to P. If the time taken by boat A is 1.5 times that of B, what is the speed of the current.

1. 10 k mph
2. 7.5 k mph ,
3. 5k mph
4. Data insufficient

Solution

If the total distance between P and Q be '2D' km . Boat A has taken more time , the direction of the current must be form Q to P if the speed of the current must be X k mph then

Time taken by A = $\frac{2d}{15-x}$

Time taken by B = $\frac{d}{15+x} + \frac{d}{15-x}$

Given $\frac{2d}{15-x}=\frac{3}{2}\left(\frac{d}{15+x}+\frac{d}{15-x}\right)$

$$\frac{4}{3}=\frac{30}{15+x}$$

Solving we get x = 7.5 k mph have option 2 is the right option.

Question

A car covers half the half distance at speed 60kmph and another half of the distance at speed 30kmph . Find the average speed of the car for the whole journey?

Solution

If total distance be '2d' then time taken to cover the first half of the journey , = $t_1=\frac{d}{60}\left(\frac{distance}{speed}\right)$

and time taken to cover the second half of the journey = $\frac{d}{30}$

Therefore total time taken = $t_1+t_2=\frac{d}{60}+\frac{d}{30}$

Avg .Speed = $\frac{total\ distance}{total\ time\ taken}=\frac{2d}{\frac{d}{60}+\frac{d}{30}}=\frac{2}{\frac{1}{60}+\frac{1}{30}}=40kmph$

Question

A car cover travels for half the time of the journey at speed 60kmph and another half of the time at 30kmph. Find the average speed of the car for whole journey?

Solution

Let the total time taken in for the journey be '2t' then distance covered in firat half of the journey = $60\times t=60t$ and distance covered in

second half of the journey $30\times t=30$

Average speed = $\frac{total\ distance}{total\ time}=\frac{60t+30t}{2t}=\frac{90t}{2t}=4$

Hence average speed of for the whole journey = 45kmph

Question

Two persons A and B walk from X to Y a distance of 27 k at 5 km/hr and 7 km/hr respectively. B reaches Y and immediately turns back meeting A at Z. What is the distance from X to Z ?

1. 25 km
2. 22.5 km
3. 24 km
4. 20 km

Solution

Let the distance from X to Z be 'a' km then time taken by X to cover distance XZ is same as time taken by B to cover distance XY – YZ = 27 + 27-a

Time Taken by A to cover XZ distance XZ = $\frac{a}{5}$

Time taken by B to cover distance XY – XZ = $\frac{54-a}{7}$

$$\frac{a}{5}=\frac{54-a}{7}$$

solving we get, a = 22.5 km

hence answer is option (2)

Equations - Simple, Special and Quadratic

Linear equation

Free Online Preparation for CAT with Minglebox e-CAT Prep. Cover basic concepts of Equations-Simple, Special and Quadratic under Quantitative Aptitude for MBA Entrance Exam Preparation with Study material, solved examples and tests prepared by CAT coaching experts.

Linear equation in one variable A linear equation in one variable is respected as ax + b = 0, where a,b are real constants and $a \neq C$. a is known as the coefficient of 'x' variable 'x' solution of linear equation in one variable is x = - $\frac{b}{a}$

any linear equation in one variable always has a solution.

Linear equation in 2 variable

Linear equation in 2 variable – Ab linear equation in 2 variable is represented as ax + by = c where a,b,c are real constants, where either a or b = 0 but not both simultaneously

equation ax + by = c has infinite number of solution for real x and y but for suitable restrictions equation may have finite number of solutions.

Simultaneous linear equations

Simultaneously linear equations in two variable

A simultaneously linear equation in 2 variable is represented as $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ (2) where $a_1, b_1, a_2, b_2, c_1, c_2$ are real constants the above pair of equation s may have none, one or infinite number of solutions depending on the relationship between the constants.

If $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{c_1}{c_2}$ then we have infinite number of solutions.

If $\frac{a_1}{b_1}\neq\frac{a_2}{b_2}\neq\frac{c_1}{c_2}$ then there is no solution

If $\frac{a_1}{b_1}\neq\frac{a_2}{b_2}$ then there is one unique solution

Equation in one variable

Equation in one variable but of higher degree. The degree of equation is the highest power of variable that exists in the equation for example

$x^3-7x+2=0$ is an equation of degree 3 in one variable .

$x^2-3x+2=0$ Is an equation of degree 2 in one variable .

An equation of second degree in one variable is known as quadratic equation.

Quadratic equation

A equation of the form $ax^2+bx+c=0$ where a,b,c are real numbers and $a\neq0$, is known as a quadratic equation .

Roots of a quadratic equation A value of 'x' say α which makes the left hand side expression equal to right hand side that is '0' .

Therefore if there exists a number α such that $a\alpha^2+b\alpha+c=0$, then ' α 'is said to be a root of the equation . In general , any quadratic will have roots either real or imaginary .
Relationship between roots of the equation and co-efficients of the equation.

If α,β are the roots of the equation and then $\alpha+\beta$ (some of the roots) = $\frac{-b}{a}\left(\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}\right)$

and $\alpha\beta$ (product of roots) = $\frac{c}{a}\left(\frac{\text{constant of } x}{\text{coefficient of } x^2}\right)$
method to solve a quadratic equation.

Completing the square method at $ax^2+bx+c=0$ be the given equation.

Step1 : add and subtract the quantity $\left(\frac{b}{2a}\right)^2$ that is

$$\left(\frac{\frac{1}{2}\times\text{coeff of } x}{\text{coeff of } x^2}\right)^2$$

Step 2 *:-* Rearrange the terms as follows

$$a\left(\left(x^2+2.\frac{b}{a}.x+\left(\frac{b}{2a}\right)^2\right)+c\frac{-b^2}{4a^2}\right)=0$$

$$\left(x+\frac{b}{a^2}\right)+\frac{4ac-b^2}{4a^2}=0$$

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Discriminant the expression b^2-4ac is known as the discrimination of the quadratic . Just by finding the discriminant we can decide the nature of the root

D>0	Roots real and distinct
D=0	Roots real and equal
D<0	Roots are imaginary
D>0 and a perfect square	Roots are distinct real and Rational

Factorization method Before attempting this method , check the discriminant of the equation . If the discriminant D is zero or a perfect square , then this method will easily given the roots of the equation . We illustrated this by an example.

Solved Examples

Question

Find the roots of $x^2 - 5x + 6 = 0$

Solution

Step i :-

Find the discriminant of the equation her by compression we get a = 1, b= -5, and c = 6 the

discriminant D = $b^2 - 4ac = (5)^2 - 4.1.3 = 25 - 24 = 1$ which is a perfect square

Step ii :-

Consider the constant term C, and factories it into 2 factors whose product is C. here C = 6

Therefore 2 factor products of 6 are

$1 \times 6, -1 \times -6, 2 \times 3, -2 \times -3$

For each pair try to see in which case the sum of factors is equal to b(the coefficient of x) here b= -5,

we see that -2×-3 satisfies this as -5 = (-2) + (-3)

Step iii :-

Write the coefficient of x as sum of the 2 factors obtained in step

So $x^2 - 5x + 6 = x^2 - 2x - 3x + 6 = C$

take out the common terms for first 2 terms and do the same for last 2 terms

x (x-2) -3 (x-2)

(x-2) is common in both the terms, so pull it out we get (x-2) (x-3) = 0

Step iv :-

Equation each term equal to '0' to get the root

x-2 =0 or x-3 =0

x-2 or x-3

Hence roots of $x^2 - 5x + 6 = 0$ are x=2 and x=3 respectively

Ratio, Proportion and Variation

Ratio is a tool for comparing 2 quantities . For example in a clear if there are 20 girls and 30 boys then we can compare the number of boys and girls as follow:

$$\frac{\text{Number of girls}}{\text{Number of boys}} = \frac{20}{30} = \frac{2}{3}$$ which is offer stated as

Number of girls : Number of boys = 2 : 3

Number of girls = $\frac{2}{3} \times \text{Number of boys}$

Number of boys = $\frac{3}{2} \times \text{number of girls}$

Ratio is a pure number it does not have a unit. The 2 numbers used in ratio are known as the 'terms' of the ratio. The first term is known as '***antecedent***' and the second term is known as '***consequent***'.

If the ratio of 2 or more quantities is given by

a:b:c

then actual quantity is given by

a*k*, b*k*, c*k*

where *k* is constant.

Proportion

Equality 2 ratio is known as proportionality that is if $\frac{a}{b} = \frac{c}{d}$ then a,b,c,d are in proportion similarly if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \text{..........} \text{ etc}$. a, b, c, d, e, f etc are in proportion.

If $\frac{a}{b} = \frac{c}{d}$ then

ad = bc

$$\frac{b}{a} = \frac{d}{c}$$

$$\frac{a}{c} = \frac{b}{d}$$

$$\frac{a+b}{b} = \frac{c+d}{d}$$

Solved Examples

Question

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{2}{3}$ then the value of $\frac{2a^2-3c^2+4e^2}{2b^2-3c^2+4f^2}$ is

Solution

As $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{2}{3}$

a = $\frac{2}{3}b$

c = $\frac{2}{3}d$

e = $\frac{2}{3}f$ putting values of a,c,ein the expression

$$\frac{2a^2-3c^2+4e^2}{2b^2-3d^2+4f^2} = \frac{2\left(\frac{2}{3}b\right)^2-3\left(\frac{2}{3}d\right)^2+4\left(\frac{2}{3}f\right)^2}{2b^2-3d^2+4f^2} = \left(\frac{2}{3}\right)^2=\frac{4}{9}$$

Question

The ratio of the current ages of Anita and Sunita is 4:3. 8 years hence, the ratio of their ages will be 6:5. then their current ages are

Solution

Let this current ages of Anita and Sunita be 4k and 3k then their ages 8 years hence will 4k+8, 3k+8 hence of their ages 8 years hence will be $\frac{4k+8}{3k+8}=\frac{6}{5}$

Solving we get 20k + 40 = 18 k + 48

2k = 8

k = 4

Hence their current ages are 16 and 12 years respectively.

Question

An engine can pull on empty boogie at the speed of 18 k mph and the reduction in its speed is directly proportional to the square root of the number of boxes it can carry of equal weight of 10 kg each. If the speed of the engine is 12 k mph when 9 boxes are loaded in the boogie. Find the maximum weight that can be carried if the speed of the engine and boogie is to be maintained at least 10 k mph ?

Solution

It is given that speed of engine and boogie without any of the boxes = 18 k mph

If 'x' is the speed of the engine and boogie with the boxes loaded , then x = $16 - k\sqrt{n}$ where k is a constant of proportionality n is the number of boxes loaded in the boogie.

From the given information we get

$$12 = 16 - k\sqrt{9}$$

k = 2

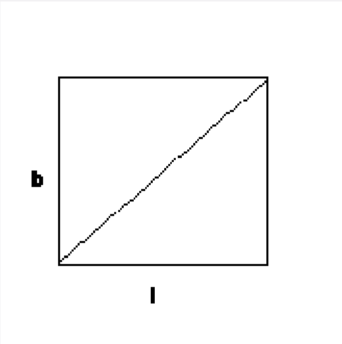
putting the maximum number of boxes allowed is 16 and therefore weight is 160 kg

Question

Instead of walking along two adjacent sides of a rectangular field a man took a shot cut along the diagonal and saved a distance equal to the longer side - then the ratio of the shortcut side to the longer side is

1. 1/2
2. 2/3
3. 1/4
4. 3/4

Solution



Let l,b the length and breath of the rectangular field then length of diagonal of the field is given by $\sqrt{l^2+b^2}$ according to given condition we get $l+b-\sqrt{l^2+b^2}=\frac{b}{2}$ (assuming l>b)

$$l-\frac{b}{2}=\sqrt{l^2+b^2}$$

Squaring both side we get

$$l^2+\frac{b^2}{4}+lb=l^2+b^2$$

$$lb=\frac{3b^2}{4}$$

$$\frac{l}{b}=\frac{3}{4}$$

Hence option (4) is the correct answer

Question

Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight . What is the weight of dry grapes available from 20 kg of fresh grapes ?

1. 2 kg
2. 2.4 kg
3. 2.5 kg
4. none of these

Solution

Let x kg be weight of dry grapes obtained from 20 kg of fresh grapes then weight of solid substance in fresh grapes will be same as in dried grapes hence we get

$$10 \times 20 = 80 \times x$$

x = 2.5 kg

Inequalities

A comparison relationship between two algebraic expression or quantities is known as an Inequalities.

For example

$3x+7 > x^2+1$

$x^2+3x+2 \geq 13x+9$ etc.

In any inequality problem variable x is assumed to be real number unless otherwise . Any inequality is generally either one of the following

'greater then' (>)

'greater than or equal to '($\geq$)

'less than' (<)

' less than or equal to' ($\leq$)

Solution of an inequalities

By solution of an inequalities we seek to find a set of value for the variable involve in the problem so that inequalities holds true for all the values lying in the solution set.

An Example

Question

3x+7 > x+1

Solution

We find that any value of x > -3 is a solution of the above inequalities. For instance

let us take x = -2, -1, 0, .5 etc. We find

3(-2)+7 > -2+1 $\Rightarrow$ 1 > -1 hence true

3(-1)+7 > -1+1 $\Rightarrow$ 4 > 0 hence true

3(0)+7 > 0+1 $\Rightarrow$ 7 > 0 hence true

Law of inequalities

if x > y then for all c x+c > y+c and x-c > y-c

If x > y and c > 0 then cx > cy and $\frac{1}{c} > \frac{y}{c}$

If x > y then for c < 0 , xc < yc and $\frac{x}{c} > \frac{y}{c}$ that is sign of inequalities gets reversed when both sides of the inequalities are multiplied or divided by the same negative quantity. If x > y >0 then $\frac{1}{x} < \frac{1}{y}$

8 > 2 > 0

$\frac{1}{8} < \frac{1}{2}$

also if 0 > x > y then $\frac{1}{x} < \frac{1}{y}$

but x> 0 > y then $\frac{1}{x} > \frac{1}{y}$

Relationship Between Power of a Number

If x > 1 then …. .... $x^3 > x^2 > x > \frac{1}{x} > \frac{1}{x^2}$ ....

but if 0< 1 then …. ..... $x^3 < x^2 < x < \frac{1}{x} < \frac{1}{x^2} < \frac{1}{x^3}$.........

In general if x > 1 then $x^m > x^n$ if m> n and if 0 < x < 1 then $x^m < x^n$ if m < n

Solving quadratic inequalities

Let us study how the sign of a quadratic expression changes as we very the value of x.

let us consider $x^2 - 3x + 2$ that is

(x-2) (x-1) for x > 2 we find x-2 > 0 and x-1>0 therefore for all x >2 (x-2) (x-1) > 0

for 1 < x < 2 x-2 < 0 but x-1 > 0 $\Rightarrow$ (x-2) (x-1) < 0

where for x < 1 then x-2 < 0 and x-1 < 0 $\Rightarrow$ (x-2) (x-1) < 0

thus if we have to solve $x^2 - 3x + 2$ > 0 we proceed as follows.

- factorize the given expression linear factors.
- Equate each linear term equal to zero and find values of x.
- Arrange the value of x is directly order , so if values are α_1 and α_2 and if $\alpha_1 < \alpha_2$ then for all $x > \alpha_2$ expression is > 0 for all $\alpha_1 < x < \alpha_2$ then expression is < 0 and for all $x < \alpha_1$ expression > 0.

While Factorizing the expression in linear factor make sure to make the sign of coefficient of x^2 as positive.

Solved Examples

Question

Solve $(x-1)^3(x+1)^2(x-3) < 0$

Solution

Observe for all $x \in R$ and for $x+1 \neq 0$, $(x-1)^2$ > 0. Therefore $(x+1)^2$ is the positive factor of the above expression hence we can divided both sides by $(x+1)^2$ we get $(x-1)^3(x-3) < 0$ also $(x-1)^3 = (x-1)^2.(x-1)$ we again observe for all $x \in R$ and for $(x-1) \neq 0$ $(x-1)^2 > 0$

Hence we just need to consider (x-1) (x-3) for solving inequalities now let's equal each linear term equal to zero we get x = 1 and x = 3 therefore for all x >3 (x-1) > 0 and (x-3) > 0 hence (x-1) (x-3) > 0 also for x < 1 , (x-1) < 0 and (x-3) < 0 hence again (x-1) (x-3) > 0 therefore solution set is given by $x \in (-\infty, 1) \cup (3, \infty)$

Question

Solve x^2 - |x| + 2 < 0

Solution

We know that $x^2 > 0$ for all x > 0

x^2 = $|x|^2$

Hence $x^2 - 3|x| + 2 < 0$ become $|x|^2 - 3|x| + 2 < 0$ factorizing into linear factor we get

$(|x| - 2)(|x| - 1) < C$

Equality each linear factor equal to zero , we get

$|x| = 2$ and $|x| = 1$

Hence if $1 < |x| < 2$ then $(|x| - 2)(|x| - 1) < 0$

but $|x| = x$ for x > 0 and

= -x for -x < 0

above solution is 1 < x < 2 or 1 < -x < -2 or -1 > x > -2 hence following we get solution is

$x \in (-2, -1) \cup (1, 2)$

Question

Solve $\frac{4}{x+2} > 4 - x$

Solution

Adding -(4 – x) to both side we get

$$\frac{4}{x+2} - (4 - x) > 0$$

$$\frac{4 + x^2 - 2x - 8}{x+2} > 0$$

$$\frac{x^2 - 2x + 12}{x+2} > 0$$

now $(x + 2^2) > 0$ for all $x \neq -2$ therefore are multiply both side by $(x + 2)^2$ we get

$$(x^2 - 2x + ´2)(x + 2) > 0$$

Discriminant of $x^2 - 2x + 12$ is given by

$$(-2)^2 - 4.1.12 = 4 – 48 = -44 < 0$$

$x^2-2x+12>0$ for all real value of 2.
dividing both sides by $x^2-2x+12$ as the solution . Hence is given by $x\in(-2,\infty)$

Question

If x > 3, y > -2 then which of the following holds good
(1) xy > -6
(2) xy < -6
(3) $x>\frac{-6}{y}$
(4)none of these

Solution

We observe that y > -2 $\Rightarrow$ y can be positive as well as negative that is if y > -2 $\Rightarrow$ y can value living between 0 > y > -2 as well as y > 0
Hence we solve this using method of options let x = 4 and y = -1
then xy = -4 > -6 therefore 'a' is true but 'b' is not also 4 > 3 hence c also holds let us take x = 5 and
y = - ½ we get

$$xy=\frac{-5}{2}>-6$$ but

$$\frac{-6}{y}=\frac{-6}{\frac{-1}{2}}=12>x=5$$

hence '3' is not trueFinally if we take x = 18 & y = -1/2 we get

$$xy=\frac{18\times-1}{2}=-9<-6$$

hence 'a' also does not s good
Hence answer is '4'

Question

Solve If x > 5 and y < -1 , hence which of the following statements is true ?
(1)(x + 4y) > 1,

1. (x > -4y)
2. (3)-4x < 5y
3. none of these

Solution

Let us go by each option and take specified values of x and y
If x = 6 and y = -10
Then x+4y = 6 + 4(-10) = -34 < 1 hence (1) is not true
also if y < 1

$-4y > 4$

but $x > 5$ therefore taking $x = 6$, $y = -10$ we get $6 < 40$ hence (2) is also not true

From (3) we find that

$x > 5$

$-4x < -20$

as $y < -1$

5y c-5

again taking $x = 6$ and $y = -10$

we find that $-24 > -50$ hence (3) is also not true

Hence answer is (4) .

Geometry

Two lines lying in a plane are said to be parallel if they never meet.

Polygons

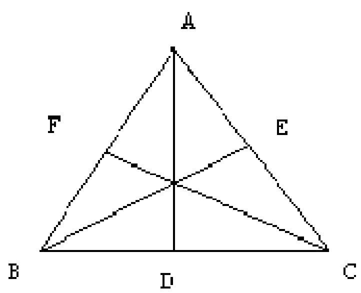
A closed figure, bounded by finite number of line segments, all lying in the same plane, is known as polygon.

Sum of interior angles of a polygon having 'n' sides, = $(2n-4)90^{\circ}$

Sum of exterior angles of a polygon = 360° (irrespective of number of sides)

Triangles

A triangle is a polygon formed by 3 line segments joined end to end.
Sum of interior angles of a triangle = 180



In any given triangle sum of 2 interior angles is equal to the third remote angle.

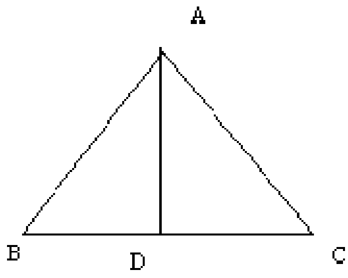
AD is the median of triangle, then $AB^2+AC^2=2(AD^2+DC^2)$

G is the centroid of the triangle .Centroid of a triangle divides the median AD in the ratio 2:1

Angle Bisector Theorem

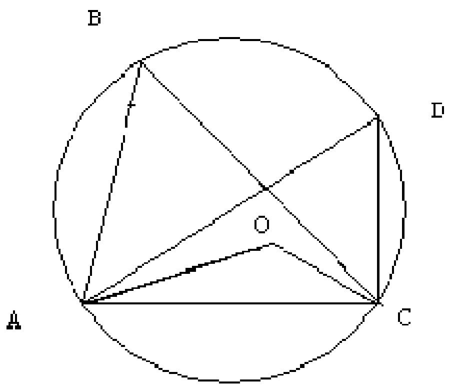
If AD is the bisector of the the angle ABC then AD divided the side BC in the ratio

$\frac{AB}{AC}$ ie $\frac{AB}{AC}=\frac{BD}{CD}$



Circle Theorem

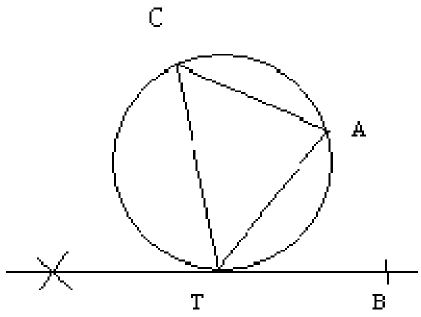
If AC is a chord of circle then angle ABC subtended by the chord at a point B on the circle is equal to



to angle subtended at point 0 lying in the same segment ie ∟ABC = ∟ADC. Also angle ABC is half of angle AOC.

Alternate Segment Theorem

If XTB is a tangent to the circle then ∟ATB =∟ACT.



Area of a triangle = $\frac{1}{2} \times base \times Height$

= $\sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$

Area of a circle of radius r = πr^2

Area of a square of side $a = a^2$

Length of diagonal of square = $\sqrt{2}a$

Area of rectangle of length l and breath b= lb

volume of cube of edge length $a = a^3$

length of cube = $\sqrt{3}a$

Volume of sphere of radius r = $\frac{4}{3}\pi r^3$

Volume of hemisphere of radius r = $\frac{2}{3}\pi r^3$

Surface area of cube = $6a^2$

Surface area of sphere = $4\pi r^2$

Surface area of hemisphere = $2\pi r^2$

Solved Examples

QUESTION

The sum of the areas of two circles, which touch each other externally, is 153 π. If the sum of their radii is 15, find the ratio of the larger to the smaller radius.

(1) 4
(2) 2
(3) 3
(4) none of these.

SOLUTION

Let the radii of the 2 circles be r_1 and r_2, then $r_1 + r_2$ = 15 (given)

and $\pi r_1^2 - \pi r_2^2$ = 153π (given)

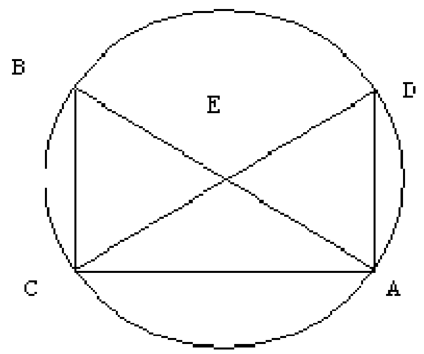
$$r_1^2 + r_2^2 = 153$$

$$r_1^2 + (r_6 - r_1)^2 = 153$$

solving we get, r_1 = 12 and r_2 = 3
ratio of the larger radius to the smaller one is 12 : 3 = 4 : 1 hence option (1) is the answer.

QUESTION

In the adjoining figure, points A, B, C and D lie on the circle. AD= 24 and BC = 12. what is the ratio of the area of Δ CBE to that of the triangle ΔADE ?

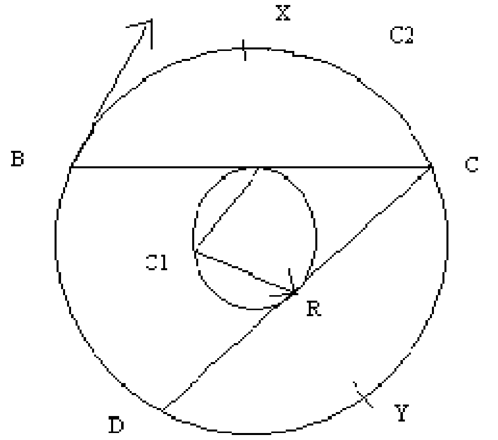


(1) 1:4
(2) 1:2
(3) 1:3
(4) data insufficient.

SOLUTION

In $\triangle CBE$ and $\triangle ADE$, ∟ CBA = ∟ CDA.
(a chord of a circle subtends equal angel at all points on the circumference, lying in the same segment)
similarly ∟ BCD = ∟ BAD and ∟ BEC = ∟ AED
Therefore $\triangle CBE = \triangle ADE$ (AAA similarity rule)
Now $\frac{BC}{DA} = \frac{12}{24} = \frac{1}{2}$
Hence $\frac{BE}{AE} = \frac{CE}{AE} = \frac{1}{2}$ hence option b).

QUESTION
C_1 and C_2 are two concentric circles while BC and CD are tangents to C_1 at point P and respectively. AB to the circle C_2, which of the following is true?



(1) $\alpha > \theta$

(2) $\alpha < \theta$

(3) $\alpha = \theta$

(4) data insufficient

SOLUTION

l (cp) = l (cr) (since tangents drawn from the same point are equal)
hence l(BC) = l(CD) (points P and R are midpoints of BC and CD as OP is ⊥BC and OR ⊥ CD)
Therefore m(arc BXC) = m(arc CYD)
hence quadrilateral OPCR is a cyclic quadrilateral.
Hence answer is option c)

But m(arc BXC) = $2\times m$ (∟ABC) = 2θ (tangent secant theorem)

therefore m(arc BZD) = $360^\circ - 4\theta$

Therefore m(BCD) = $\frac{1}{2}\times m(\text{arc BZD}) = 180^\circ - 2\theta$

Hence m∟PCR = $180^\circ - 2\theta$

m∟POR = 2m ∟PQR = 2α (angle at center)

m∟OPC + m∟ORC = 180°

Hence m∟POR + m∟PCR = 180°

Therefore $2\alpha + 180^\circ - 2\theta = 180^\circ \Rightarrow \alpha = \theta$

QUESTION

Two circles of radius 3 units and 4 units are at some distance such that length of the transverse common tangents and the length of their direct common tangents are in the ratio 1: 2 . What is the distance between the centres of those circles?

(1) $\sqrt{50}$ units

(2) $\sqrt{65}$ units

(3) 8 units

(4) cannot be determined .

SOLUTION

Let x = distance between the centres of the circles.

T= length of the transverse common tangent.

D = length of direct common tangent.

$\Rightarrow t^2 = x^2 - (r_1 + r_2)^2$

And r_1 = 4 units and r_2 = 3 units

then, t = $\sqrt{x^2 - (r_1 + r_2)^2}$ --------------- 1)

also d = $\sqrt{x^2 - (r_1 - r_2)^2}$

$x^2 - d^2 = (r_1 - r_2)^2$ -------------------- 2)

$= 4 \times 4 \times 3 = 48$

Since $\frac{d}{t} = \frac{2}{1}$

$48 = 4t^2 - t^2 \Rightarrow t^2 = 16 \Rightarrow t = 4$ units

hence $x^2 = (4-3)^2 + 8^2 = 65$

hence x = $\sqrt{65}$

from 1) and 2) we get

$x^2 - d^2 = 4r_1r_2$

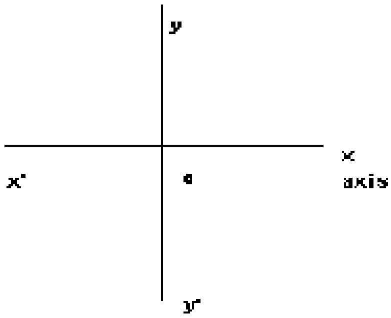
d = 2t

d = 8 units

hence answer is option 2).

Co-Ordinate geometry

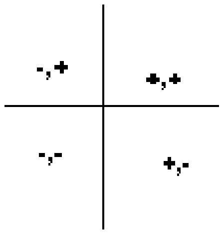
In two dimensional Coordinate Geometry, location of any point lying in the plane, is given by specifying the perpendicular distances of the point, from a set of fixed mutually perpendicular lines. The fixed mutually perpendicular lines are known as X-axes and Y-axes respectively. The point of intersection is known as the origin 'O'.



These 2 lines divide the given plane in 4 parts known as quadrants. Distances measured to the right hand side of origin O are treated as positive and distances measured towards the left of origin are treated as negative. In a similar way distances measured along the Y-axis and above the X-axis are treated as positive distance measured below the X-axis are treated as positive distances measured below the X-axis are treated as negative.

The Coordinates of a point are specified as an ordered pair, Comprising of its distance measured along the X and Y axis . Distance measured along one X-axis from the origin is known as abscissa, and usually denoted X, the G distance measured along the Y-axis From the origin is called the ordinate of the point and is denoted by y. Thus coordinate is of a point are specified as (x,y) I.e as (abscissa, ordinate).

The 4 quadrants are known as 1 st ,2 nd 3 rd & 4 th quadrant. The below diagram summarizes the sign of the abscissa and the ordinate of x points lying in 1 st , 2 nd , 3rd , 4 th quadrant.

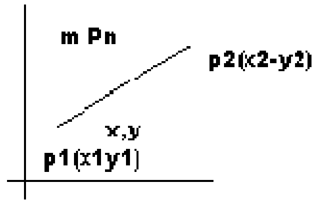


Distance Formula

The distance d between 2 points P_1 and P_2 having coordinates (X_1, Y_1) and (X_2, Y_2) is given by

$$D = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

Section formula and mid point formula



The coordinates P (x,y) of a point which divides the join of points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ in the

ratio m:n is given by X = $\frac{mx_2+nx_1}{n+n}$; Y = $\frac{my_2+ny_1}{m+n}$

Mid point Formula

The Coordinates of the mid point of the line joining $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by X =

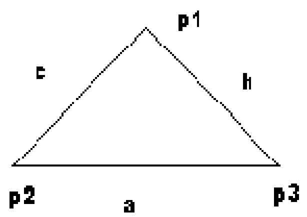
$\frac{x_1+x_2}{2}$; Y = $\frac{y_1+y_2}{2}$

Coordinates of the centroid of a triangle

Let $P_1(x_1,y_1)$, $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$ be the coordinates

of the vertices of triangle $P_1P_2P_3$, then coordinates of it's centroid (x,y) is given by X =

$\frac{y_1+y_2+y_3}{3}$; Y = $\frac{y_1+y_2+y_3}{3}$



The coordinates of the incentre I (x,y) of the triangle $P_1P_2P_3$ with vertices $P_1(x_1,y_1)$; $P_2(x_2,y_2)$

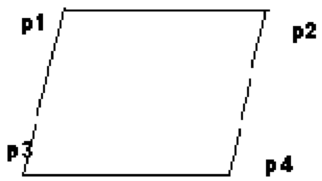
$P_3(x_3,y_3)$ and side length a,b,c is given by X = $\frac{ax_1+bx_2+cx_3}{a+b+c}$ $\frac{ay_1+by_2+cy_3}{a+b+c}$

Area of a triangle

The area of triangle having vertices $P_1(x_1,y_1)$; $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$ is given by = $\frac{1}{2}$ [$(x_1y_2-x_2y_1)$ + $(x_2y_3-x_3y_2)$ + $(x_3y_1-x_1y_3)$]

Note – If area of is Zero $\Rightarrow$ points P_1, P_2, P_3 are collinear.

Condition for 4 points , no three of which are collinear to be a parallelogram . Let $P_1(x_1, y_1)$ $P_2(x_2\ y_2)$, $P_3(x_3, y_3)$, $P_4(x_4, y_4)$ be 4 points lying in a plane then P_1 , P_2 , P_3 , P_4 are the verticals of a parallelogram if $x_1+x_3=x_2+x_4$ and $y_1+y_3=y_2+y_4$

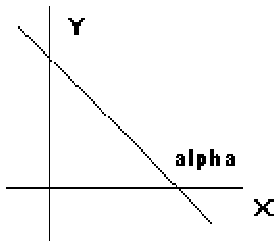


P_1, P_2, P_3, P_4 are the vertices of a square if in addition to above or $x_4-x_1=y_3-y_2, y_4-y_1=x_3-x_2$

Equation of a Line

Slope of a line

The slope of a line is defined as tangent of the angle α which the line makes in the positive direction of the x-axis in the anti – clockwise direction.



The slope is generally represented by 'm' thus in the notion used above $m=\tan\alpha$

The equation of a line is given by y = mx+c where 'c' is the intercept mode on the y -axis by the line.

Equation of x-axis is y = 0

Equation of y-axis is x = 0

2 lines are said to be parallel if they have the same slope that is if where $m_1=m_2$ where m1 and m2 are the slopes of the 2lines.

2 lines said to be perpendicular if they product of there slope is -1. that is if $m_1 . m_2=-1$ then lines are perpendicular to each other. Different forms of the equation of a line.

Line passing through point $P(x_1, y_1)$ and having slope 'm' . $(y-y_1)=m(x-x_1)$

Line passing through points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

If a line makes intercept a,b on x and y-axis respectively then equation of line is given by $\frac{x}{a}+\frac{y}{b}=1$

Angle between 2lines having slopes m_1 and m_2 is given by $\theta=\tan^{-1}\left(\left|\frac{m_1-m_2}{1+m_1m_2}\right|\right)$ where θ is the actual angle between the 2 lines.

Perpendicular distance between parallel lines , Let the equation of 2 lines be $y=mx+c_1$ and line is given by $\left|\frac{c_1-c_2}{\sqrt{1+m^2}}\right|$

Perpendicular distance of a point from a line . Let $p(x_1,y_1)$ be any point and let y = mx +c be any line, then the perpendicular distance of point p from line y = mx + c is given by $\left|\frac{y_1-mx_1-c}{\sqrt{1+m_2}}\right|$

Equation of a Circle

The equation of a circle having center at point (h,x) and radius 'r' given by $(x-h)^2+(y-x)^2=r^2$

Solved Examples

Question

Find the equation of the line with slope 2 and intercept on the y-axis as -7 ?

Solution

we know that equation of the line having slope 'm' and intercept on y-axis is 'c' the equation is y = mx +c , hence equation of line is y = 2x – 7

Question

The equation of a line which makes an intercept of 3 on x-axis and -3 on y-axis is

1. x – 2y = 5
2. x – 2y = 3
3. 4x + 13y = 7
4. none of these

Solution

The line with the x-axis at point (3,0) and y-axis at the point (0,-3) hence equation of line is using

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y - 0 = \frac{-3-0}{0-3}(x-3)$$

y = x-3 or x – y = 3

hence answer is option (2)

Question

Find the equation of the line perpendicular to the line x + y = 2 and passing through (1,2)

1. y = x + 3
2. y = x – 1
3. y = x + 1
4. y – x = 2

Solution

Let slope of required line be 'm' then slope of line x + y = 2 is '-1' for lines to be perpendicular the product of their slope should be '-1' hence we get

$$m \times (-1) = -1$$

m = 1

using one point slope from the equation of we get required equation as

(y – 2) = 1. (x-1)

y = x + 1

Hence (3) is the answer.

Question

The equation of the line through the intersection of the lines 2x + y = 3 and 3y – x + 2 = 0 and having slope -1/2 is

1. 14y +5x +10 = 0
2. 14y+7x-9 = 0
3. 14y+7x-6 = 0
4. 14y+7x+11 = 0

Solution

Equation of any line through the point of intersection of the given lines is of the form (2x+y-3) + (3y-x+2) = 0 where k is the constant to be determined , remaining we get

(3k+1)y + (2-k)x+2k-3 = 0,but slope of this line $\frac{k-2}{3k+1} = \frac{-1}{2}$ (given) solving we get k = 3/5

hence putting the value of k = 3/5 we get equation required line as 14y+7x-9 = 0

Question

The coordinate of the verities of a triangle are

1. (3,5)

2. (2,6)
3. (4,4)
4. (2,4)

Solution

The Coordinate of the centroid of the triangle with vertical $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by X = $\frac{x_1+x_2+x_3}{3}$: Y = $\frac{y_1+y_2+y_3}{3}$

Hence X = $\frac{2+3+1}{3}=2$ Y = $\frac{3+5+4}{3}=4$

Therefore option (4) is the correct answer

Permutations & Combinations

Permutations

Free Online Preparation for CAT with Minglebox e-CAT Prep. Cover basic concepts of Permutations and Combinations under Quantitative Aptitude for MBA Entrance Exam Preparation with Study material, solved examples and tests prepared by CAT coaching experts.

The number of arrangements of 'n' distinct object into 'n' distinct position is given by n,n-1,n-2,........3,2,1, which is often denoted as n! (read as n 'factorial'). The number of arrangements is also know as “Permutations". Thus number of permutation of n distinct object into n distinct places is denoted as P_n^n , read as permutations of n distinct objects taken n at a time.

The number of Permutations of n distinct objects taken from a group of 'r' distinct objects, in n distinct place is given by $P_r^n = \frac{n!}{(n-r)!}$

The number of Permutations of n object out of which m are of one type, q are of second type etc. Such that n ≠ m ≠q .... is given by $\frac{n!}{m!.q!....}$

The number of Permutation of n distinct objects taken all as times around a circle is given by (n-1)!

And $\frac{(n-1)!}{2}$ accordingly as clockwise and anticlockwise arrangements are treated as different or same. This formula is valid for a necklace.

Number of arrangements of n objects, such that any p out of those occur together is given by

(n-p+1) ! * p !.

Number of arrangements of n distinct things, when any object can be repeated any number of times is

given by n^r .

Combinations

Number of combinations of r distinct things taken out of n distinct things is given by

$$C_r^n = \frac{n!}{(n-r)!r!}$$

Please note in case of combinations also known as selections the order in which things are chosen is not important. Thus if we consider 4 distinct alphabets viz. A,B,C & D then no of permutations of any 3 alphabets out of is given by

ABC

ABD

ACB

ADB

ACD
ADC, etc whereas no. of selections is ABC,ABD,ACD
and BCD i.e. 4 only.

Number of ways of selecting r things out of n distinct things is same as number of ways of selecting n-r things out of n things. i.e.

$$C^n_r = C^n_{n-r}$$

$$C^n_r = C^n_{r-1} + C^n_{r+1}$$

No of ways in which none or some objects can be chosen out of n distinct of objects is given by

$$2^n = C^n_0 + C^r_1 + C^n_2 + \ldots\ldots C^n_n$$

Distribution

The total number of ways in which n identical things can be distributed into r distinct boxes, such that one or more than one box may remain empty, but not all the boxes can be empty is given by

$$C^{n+r-1}_{r-1}$$

If blank boxes are not allowed i.e each box has at least one object then no of ways is given by

Number of non-negative solutions of is given by

$$C^{1-1}_{\cdot-1}$$

Number of Non – negative solution of $x_1 + x_2 \ldots\ldots\ldots\ldots x_r = n$ is given by

$$C^{n+r-1}_{r-1}$$

Solved Examples

Question

A 3 digit number is formed using the digits 2,3 and 4 without repeating any one of them what is the sum of all such possible numbers ?

Solution

Let us first find the total number of 3 digit numbers which be formed using the digits 2,3 and 4. The total number of such numbers is 3 ! = 6 now out of these 6 numbers we notice that number 2 appears in the hundreds place as well as in tens place and units place. So let us see how many numbers are there where 2 appears in the hundreds place we find there are exactly 2! numbers. Similarly numbers 3 and 4 appear 2 ! times in hundreds place. Hence if split each of the 6 numbers as say for example 234 as

$2\times100+3\times10+4$ then if we add all the numbers we see that numbers 2,3,4appear in hundreds place 2! times each. Sum of hundreds digits of all the numbers – Similarly sum of all the $(2+3+4)\times2!\times100$ numbers in the tens and units digit is given by $(2+3+4)\times2!\times10$ and $(2+3+4)\times2!.1$ therefore sum of all the numbers is

$(2+3+4)\times2!\times(100+10+1)$

= $9\times2!\times111$

= 1998.

Question

Atul has 9 friends ; 4 males and 5 females. In how many ways can he invite them, if he wants to have exactly 3 females in the invites ?

Solution

The 3 girls which are to be invited can be selected in C_3^5 ways. Also the remaining 4 males none, all or some can be invited in 2^4 ways. Hence Total no of ways Atul can invite his friends is

$C_3^5\times2^4=10.16=160$

Question

How many numbers can be formed from 2,3,4,5,6 (without repetition), when the digit at tens place must be greater than that in the hundreds place ?

Solution

Total number of numbers which can be formed using the digits 2,3,4,5,6 only once (without repetition) is 5! = 120. Now since digits are not be repeated hence if we consider any number out the 120 so formed, then either the tens digits will be greater then hundreds digits or smaller than it. Hence we have 2 possibilities for any number. Therefore half the numbers out of 120 will satisfy this and other half will not. Hence answer is 60.

Question

A polygon has 35 diagonals find the number of sides of the polygon.

Solution

The number of diagonals of a polygon having n sides is given by $\frac{n(n-3)}{2}$. But no of diagonals is 35

$$\frac{n(n-3)}{2} = 35$$

$$n(n-3)= 70$$

Solving we get n = 10. Hence number of sides of polygon is 10.

Progressions

A progression or sequence is defined as a succession of terms arranged in a definite according to some rule.
For example the sequence of Odd numbers
1,3,5,7,9,11,etc......

The numbers in the sequence are called the terms of the sequence. A sequence having a finite number of terms is known as finite sequence.

The first terms in a sequence is generally denoted by a, T_1 second term by a_2, a_3.......... or T_2, T_3........etc . The n^{th} term of the sequence is denoted by a_n or T_n and is also known as the general term of the sequence, as by assigning value to 'n' , T_n can be made to represent any of the terms in the sequence.

Example
If the general term of a sequence is given by 2n, Find the First, sixth and 8 th term.

Given Tn = 2n
Therefore
First Term $\Rightarrow$ n=1
Therefore T_1 =2.1 = 2
Similarly T_6 = 2.6 = 12 and
T_8 = 2.8 = 16

Some Standard type of sequences:

Arithematic Progression (AP)

In this type of sequence, the difference between any two consecutive terms of the sequence is a constant. The constant is known as the 'Common difference.' Thus if the first term in AP is a, and Common difference is 'd' then second term is given by T_2 = a+d, third terms T_3 = a+2d etc.....
in general T_n = a+(n-1)d.

Sum of n terms of an AP is given by $Sn=\frac{n}{2}[2a+(n-1)d]$

Also $Sn=\frac{n}{2}[T_1+Tn]$

Geometric Progression (GP)

In this type of sequence the ratio of 2 consecutive terms is a constant. The constant ratio is know as common ration and is usually denoted by 'r'. Thus if 'a' is the first terms of a GP
then T_2 = ar , T_3 = ar^2 etc.

Sum of n terms of a GP, $S_n=\frac{a(r^n-1)}{r-1}$ if $r>1$

$=\frac{a(1-r^n)}{1-r}$ if $r<1$

Sum up to infinite terms of a GP

If $r<1$ then r^n -------->0 as n-----> ∞ i.e. if lets say $r=\frac{1}{2}$ then if we consider higher and higher values of n then r^n becomes smaller and smaller. Hence if we consider the sum up to very large number of terms, we say the sum up to infinite terms, $S_\infty=\frac{a}{1-r}$

Harmonic Progression (HP)

A sequence a_1,a_2,a_3......... is said to be a Harmonic Progression

if $\frac{1}{a_1},\frac{1}{a_2}$..... are in AP.

The n^{th} term of HP is given by $T_n=\frac{1}{a+(n-1)d}$

There is no formula for finding the sum up to n terms of a HP.

Arithmetic mean (AM) of 2 positive numbers a,b is defined as $AM=\frac{a+b}{2}$

Geometric mean (GM) of 2 positive numbers a,b is defined as $GM=\sqrt{ab}$

Harmonic mean (HM) of 2 positive numbers a,b is defined as $HM=\frac{2ab}{a+b}$. For any given a,b $AM\geq GM\geq HM$

Solved Examples

Question

Find the 10 th terms of the series: $5,\frac{7}{2},\frac{7}{4},\frac{9}{8}$.......

(1) $\frac{-9}{512}$

(2) $\frac{-15}{512}$

(3) $\frac{-1023}{512}$

(4) $\frac{-2559}{512}$

Solution

Term of the series can be split as $4+1, 3+\frac{1}{2}, 2+\frac{1}{4}$.......
We find 4,3,2....... are in AP with common difference of -1 hence 10 th term is
given by T_{10} = 4 + (10-1)(-1) = -5
[as T_n = a +(n – 1) d]
Also $1, \frac{1}{2}, \frac{1}{4}$ are in GP
Hence T_{10} = $1.\left(\frac{1}{2}\right)^{10-1} = \frac{1}{2^9}$
[as $T_n = a.r^{n-1}$]
Hence T_{10} = $-5+\frac{1}{2^9}$ = $\frac{-2559}{512}$

Question

If the 8 th term of an AP is 88 and the 88 th term is 8, then 100 th term is :
(1) 1
(2) 2
(3) -4
(4) 8

Solution

Given 8 th = 88 and 88 th term = 8, let the first term of AP be 'a' and common difference be 'd'.
then $T_8 = a+7d$ and $T_{88} = a+87d$
⇒ 88 = a + 7d (1)
and 8 = a + 87d (2)
Subtracting we get 80 = -80d
⇒ d = 1
and putting (1) we get a = 95
Hence,
T_{100} = a + 99d
= 95 + 99(-1)
= -4
Hence option (3)

Question

In the AP 3,7,11,15..... up to 50 terms and 2,5,8.....up to 50 terms, how many terms are identical ?
(1) 12
(2) 4
(3) 16
(4) 18

Solution

For the AP 3,7,11,15....
$a_1 = 3$ and $d_1 = 4$
$T_n = 3 + (n-1)4 = 4n - 1$
and for the AP 2,5,8,.......
$a_2 = 2$; $d_2 = 3$
$T_m = 2 + (r-1)3 = 3m - 1$
if n^{th} terms of 1st AP is equal to the m^{th} term of Second AP, then
4n-1 = 3m-1
$\Rightarrow$ 4n = 3m or 4n = 3m
$\Rightarrow$ $\frac{n}{3} = \frac{m}{4}$ = x Say.
As $n \leq 50$ and $m \leq 50$
Hence $x \leq \frac{50}{3}$ and $x \leq \frac{50}{4}$
As x has to be an integer $x \leq 12$
For x = 1,2,3......12
Thus there are 12 terms in the 2 sequences which are equal.

Question

The product of the first terms of the G.P having the third term as 24 as -
(1) 16
(2) 32
(3) 64
(4) 128

Solution

Let the third term be 'a' then first, second fourth and fifth term will be given as
$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$ where r is the common ratio.

Now product of first five terms = $\frac{a}{r^2}.\frac{a}{r}.a.ar\ ar^2$

= a^5

but a = 2

⇒ product = 2^5 =32

Hence (2)